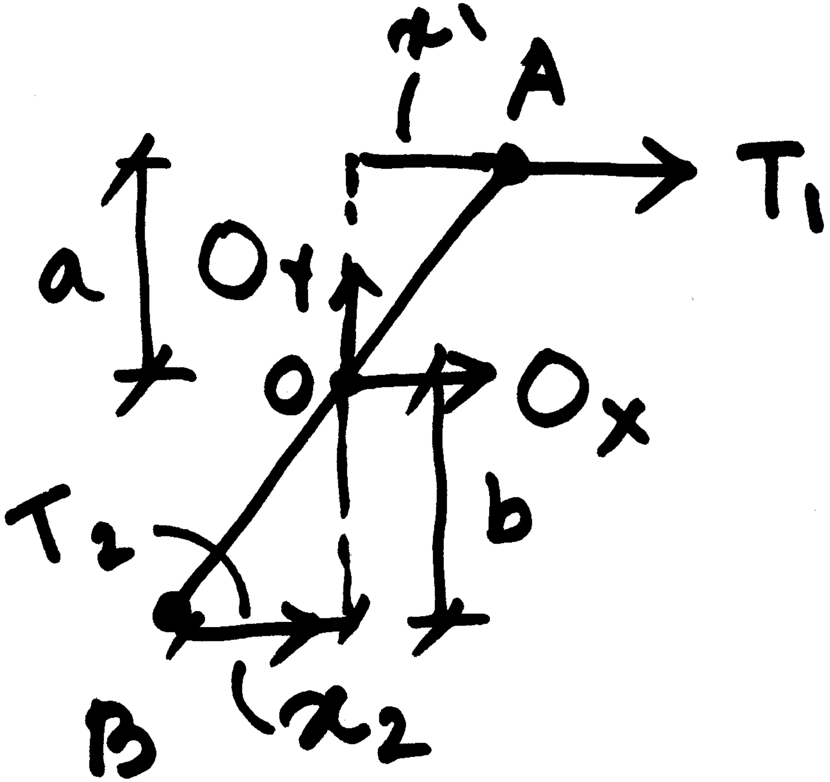
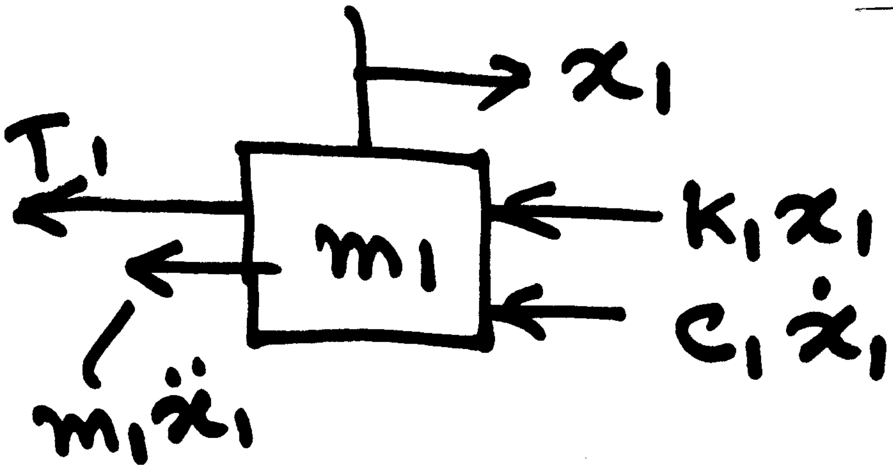
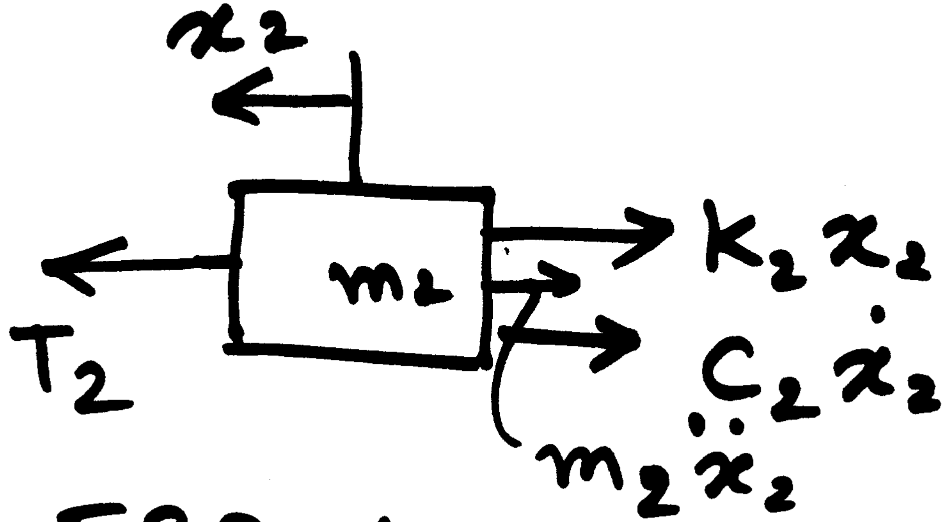
Prof. D. Choudhury
Recap lecture for Lec-4.
01/3/13



FBD of link AB



FBD of m1



FBD of m2

From similar Δs,

$$\frac{x_1}{a} = \frac{x_2}{b} \quad \text{--- (1)}$$

$\therefore$ It is Single DOF. Ans.

FBD of link AB,

$$\Sigma M_0 = \cancel{I_0} \, 0 \quad (\because \text{mass negligible})$$

$$\therefore T_1 a - T_2 b = 0 \quad \text{--- (2)}$$

FBD of m_1

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 + T_1 = 0 \quad \text{--- (3)}$$

FBD of m_2

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 - T_2 = 0 \quad \text{--- (4)}$$

From (3), using (2).

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 + \frac{T_2 \cdot b}{a} = 0 \quad \text{--- (5)}$$

Using (4),

Eq. (5) becomes,

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + \frac{b}{a}\left(m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2\right) = 0$$

From, (1),

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + \frac{b}{a} \cdot \frac{b}{a}\left(m_2 \ddot{x}_1 + c_2 \dot{x}_1 + k_2 x_1\right) = 0$$

$$\Rightarrow \left(m_1 + \frac{b^2}{a^2} m_2\right) \ddot{x}_1 + \left(c_1 + \frac{b^2}{a^2} \cdot c_2\right) \dot{x}_1 + \left(k_1 + \frac{b^2}{a^2} \cdot k_2\right) x_1 = 0 \qquad (6)$$

Using, $m_1 = m_2 = m$

$k_1 = k_2 = k$

$c_1 = c_2 = c$

Eq. (6), becomes,

$$\left(1+\frac{b^2}{a^2}\right)(m\ddot{x}_1 + c\dot{x}_1 + kx_1) = 0$$

or, $$\boxed{m\ddot{x}_1 + c\dot{x}_1 + kx_1 = 0} \quad \left(\because 1+\frac{b^2}{a^2} \neq 0\right)$$

Ans

Natural circular frequency,

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90}{10}} = 3 \text{ rad/s}$$

Natural period,

$$T_n = \frac{2\pi}{\omega_n} = 2.1 \text{ sec.} \quad \underline{\text{Ans.}}$$

$\therefore f_n$ = Natural frequency,

$$= \frac{1}{T_n} = 0.478 \text{ Hz.}$$

Critical Damping,

$$C_c = 2 m \omega_n$$

$$= 2(10)(3) \ N.s/m$$

$$= 60 \ N.s/m.$$

Damping Ratio $= \eta = \dfrac{c}{C_c}$

$$= \frac{6}{60} = 0.1 \text{ or } 10\%$$

Damped circular frequency

$$\omega_d = \omega_n \sqrt{1-\eta^2}$$

$$= 3\sqrt{1-(0{\cdot}1)^2} \text{ rad/s}$$

$$= 2{\cdot}985 \text{ rad/s}$$

$\therefore$ Damped period $T_d = \dfrac{2\pi}{\omega_d} = 2{\cdot}104$ sec.

Damped frequency $f_d = \dfrac{1}{T_d} = 0{\cdot}475$ Hz

Ans